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THE ECONOMICS OF BUILDING CODES TO RESIST SEISMIC SHOCK

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ABSTRACT

This paper applies the theory of optimal investment under risk to the problem of evaluating building codes to make structures more resistant to earthquakes. A simple equation is derived that can be used to estimate the implicit values of the social costs of earthquake damages that are necessary to justify an increment to the seismic resistivity of a structure, and an illustrative empirical application is made to evaluate a recently proposed earthquake building code. The paper also examines the effects of earthquake prediction information on both private decisions regarding the structural integrity of buildings and the social attractiveness of seismic building codes.

THE ECONOMICS OF BUILDING CODES TO RESIST SEISMIC SHOCK

Linda Cohen and Roger Noll*

The purpose of this paper is to apply economic analysis to the problem of evaluating building codes that are designed to mitigate the damaging effects of earthquakes. Earthquake damage is a probabilistic event, and the best technology for mitigating earthquake damage entails increasing the capital costs of threatened structures. Consequently, selecting an optimal seismic resistivity for buildings is a problem in optimal investment planning when returns are risky. Most types of disasters and defenses against them also have these general characteristics, and the methods used in this paper apply to evaluating defenses against all such disasters, ranging from flood control projects to emergency core cooling systems for nuclear reactors. Nevertheless, each form of disaster has unique technical characteristics that are of potentially great empirical importance. Hence, this paper eschews generality and deals explicitly with the problem of selecting optimal building standards in seismically active areas.

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For the most part, seismic codes apply to new structures. The codes specify minimum design features that new structures must match or beat. The codes are designed so that a conforming structure will be able to withstand a specified intensity of violent ground motions without collapsing, although building to code also diminishes the damage to the structure from earthquakes that are not severe enough to cause the building to collapse. In some instances, new information about the vulnerability of structures or the nature of the threat of an earthquake in a particular area leads local governments to adopt codes for existing buildings as well as new structures. For example, the Field Act in California, passed after the damaging Long Beach earthquake of 1934, required that all schools in the state, old and new, be made to withstand a major earthquake. In the aftermath of the 1971 San Fernando earthquake, measures have been adopted or proposed that would require retrofitting of particularly hazardous structures, such as dams, public meeting halls and theaters, to bring them up to the standards required of new structures.

The primary economic justification for seismic building codes is that the structural soundness of a building has a social value that is not likely to be taken into account by its owner.—/

/ The analysis in this paper ignores arguments for building codes and other public policies related to disasters that presume an inability of humans to calculate accurate probabilities of disasters or to behave rationally when confronted by low-probability, catastrophic contingencies.

If a building collapses during an earthquake, the owner suffers a financial loss equal to the value of the structure. But the collapse of a building can have a higher social cost than simply its asset value. First, occupants of the building or persons in its immediate vicinity may be killed or maimed by the collapsing structure. Second, other capital assets, such as adjacent buildings or vehicles, may be damaged by its collapse. Third, government resources are used to clean up part of damage of an earthquake and to retain order in damaged areas.

The owner of a building lacks the incentive to consider the full social cost of the collapse of a building, even if the owner is liable to compensate persons who suffer damage when the building collapses. For several reasons, complete compensation is unlikely to be paid, and so the owner will not be led to minimize the sum of the costs of earthquake defenses and liability for earthquake related damages. First, the collapse of a building may bankrupt its owner. In any event, the wealth of the owner, whether an individual or a corporation, represents the legal limit to compensation. Second, since an important element of the damage of an earthquake is loss of life and serious injuries, compensation is likely to be, at best, arbitrary, if not systematically too small. Third, whatever their merit as policies, governmental disaster relief programs are, in effect, social insurance policies that pay a substantial part of the costs of a disaster. These programs affect incentives to defend against disasters, creating a type of moral hazard problem.—/ Fourth,

—/ Linda Cohen, "A Public Policy Approach to the Study of Optimal Compensation Systems: the Case of the Price-Anderson Act," Kennedy School of Government Discussion Paper No. 61D, Harvard University, February 1979.

in a major earthquake the cause of a particular part of the damage is likely to be difficult if not impossible to ascertain. If a block of buildings collapses, it is difficult to determine which would have withstood the earthquake if others had not collapsed, and which individuals and vehicles in the surrounding area were damaged by which building.—/

/ This paper does not systematically examine taxation according to the seismic vulnerability of a structure as an alternative to codes. Obviously, basing taxes in part on seismic risk categories may be a desirable policy, although it has certain problems owing to the uncertainties associated with the likelihood an earthquake will occur, the actual seismic resistivity of any particular method of construction before a structure has been subjected to seismic shock, and the fact that earthquake damage is sometimes difficult to attribute to any particular building or structural feature.

Much of the damage associated with earthquakes is so-called secondary damage -- that is, destruction caused not by the earthquake directly, but by other problems that arise because of the earthquake. For example, more than ninety percent of the damage and deaths

associated with the 1906 San Francisco earthquake were directly attributable to the ensuing fire.—/ If the collapse of one building

/ Charles Boden, "San Francisco's Cisterns," California Historical Quarterly, Vol. 15, No. 4 (1937), pp. 1-13.

is accompanied by fire, at what point does fire damage cease to be the responsibility of the owner of the collapsed structure and begin to be the responsibility of the local fire department? In New York State, for example, in the case of a fire resulting from negligence, liability is limited to damages to the structures that are adjacent to the structure in which the fire started. Owners of all other damaged structures have no claims against the negligent party or owners of structures through which the fire passed.—/ Although there are

/ Marc Franklin, Tort Law and Alternatives: Injuries and Remedies, The Foundation Press, Inc., Mineola, New York (1971), pp. 170-171.

economically sound historical reasons for this policy, its perverse incentives vis-a-vis earthquake resistant construction practices are clear.

In sum, determining who is liable for the damages suffered by each person as a result of an earthquake is an all but impossible task, and probably fruitless in any event because some who are ultimately held responsible for the damage will be unable to compensate the victims. Seismic building codes are one mechanism for dealing with this problem. A properly designed code can effect an approximate internalization of the social costs of earthquake damage.

The next section develops a simple theoretical model of the choice of an optimal building code, given that differing codes imply differing cost increments for structures and provide differing degrees of protection from seismic shock. The purpose of the section is to derive equations that can be estimated using the limited data now available. Several important simplifying assumptions are employed; these are relaxed in Section III.

The second section examines the rudimentary data that are available to examine the extent to which building codes might be said to be economically optimal. Because the benefits of codes include savings in human lives and injuries that are difficult, if not impossible, to evaluate, no definitive judgment on codes is offered. Instead, the second section contains a calculation of the magnitude of these benefits that would rationalize existing and proposed codes.

The third section provides a theoretical analysis of the effects of changes in the expected frequency of earthquakes on decisions about optimal seismic resistivity. This analysis is useful for gaining insights about the likely effect of the development of techniques for predicting earthquakes.

I. A SIMPLE THEORETICAL FRAMEWORK

The problem of devising an optimal seismic building code is regarded here as equivalent to a problem of minimizing the costs of a long-lived capital investment, including the expected damage of earthquakes. The extent to which this random event undermines the value of the asset depends upon the amount of defensive expenditure

made at the time the investment was made.

Henceforth, the following notation will be adopted:

K = investment in the structure that yields income, including the contents of the structure.

x = the additional investment in a structure for the purpose of increasing its resistivity to seismic shocks, measured as a percent of income-earning investment K (the total investment in the structure is $(1 + x)K$).

$p(t)$ = the probability that an earthquake will occur at time t that causes structural collapse if $x = 0$.

i = a measure of the intensity of ground shaking associated with earthquakes.

r = the market rate of interest.

v = the rate of depreciation of the structure.

N = the useful life of the structure.

$q(i)$ = the probability that shaking of intensity i will occur at the site of the structure, given that a major earthquake has occurred in the area.

$f(K, x, i)$ = the proportion of the income-earning investment in the structure, K , that would be destroyed by ground shaking of intensity i if defensive expenditures xK have been made on the structure.

The damage function, f , is assumed to depend on K ; that is, a given percentage increment in costs for a structure has a different protective effect on buildings of differing costs. In general,

larger structures are more likely to be damaged by an earthquake. In reality, of course, other structural features of a building, such as height and construction materials, determine in part the relationship between the amount of damage suffered and the expenditures on defense against seismic shocks. Nevertheless, as a general proposition, bigger structures are more prone to damage given any intensity i (e.g. $f_K > 0$) and benefit more from defensive expenditures ($f_{Kx} < 0$). Since $f(K, x, i)$ must be bounded above by one and below by zero, at least for large values of K and x , $f_{KK} \leq 0$ and $f_{xx} \geq 0$.

The earthquake probability function, $p(t)$, would normally be estimated from historical frequencies of damaging earthquakes, although in the future reliable earthquake predictions may be possible. Because damage from ground motion diminishes as one moves away from the center of an earthquake, $p(t)$ varies from location to location.

The characterization of ground shaking used in this model is greatly simplified from reality, but is sufficient to capture the policy problem at issue. The function $p(t)$ is the p.d.f. that an earthquake of sufficiently large magnitude to impose uncompensated losses will occur at time t . Because the principal external costs of earthquakes arise only when structural collapse occurs, small earthquakes which cause some private losses are ignored. Moreover, earthquakes are assumed here to be sufficiently infrequent that at most one damaging earthquake will occur during the planned life of the structure.

The expected life of a structure is assumed to be independent of the amount of seismic resistivity built into it.

In reality, the type of reinforcing that makes a building more earthquake resistant also increases its durability. If N is large, as is the case for nearly all buildings, the discounted present value of the income stream added because of the building code is small, and the analysis is greatly simplified if it is ignored.

The problem of picking an optimal building code is regarded as equivalent to picking an optimal value of x , the proportion of the capital investment that is attributable to additional seismic resistivity over that which would be built into the structure inadvertently, even if the structure were located in an area with no seismic risk. Of course, a modern structure is naturally resistant to earthquakes because design features that add to its longevity, insulation from wind resistance (in the case of tall structures), and other desirable structural characteristics also make the building somewhat resistant to seismic shock. The variable x measures expenditures specifically for the purpose of providing added seismic resistivity.

This particular formulation of the problem presumes that building codes are suboptimized, i.e., that they specify the maximal seismic resistivity available at the cost of implementing the codes. For most types of building codes, this would be a facetious assumption, because the point of many codes is to protect particular product and labor inputs; however in the case of building codes in seismically threatened areas, the assumption probably is not violently wrong. Academic structural engineers play a major role in designing and evaluating seismic codes, and receive no

economic benefits from the effects of the codes on the choice of building inputs. Moreover, the codes are based in part on experimental and theoretical engineering research that is widely published in the professional literature, and subject to disinterested professional scrutiny. Thus, it is not likely that, for any given degree of seismic resistivity contemplated in a code, an alternative technology is readily available that could provide the same protection at significantly lower cost. Of course, it may still be the case that the extent of seismic resistivity that is implicit in the code is not optimal.

The firm's optimal choice of x is that value that minimizes private costs, i.e. the sum of the initial investment and expected earthquake damage. The latter is the depreciated cost of the investment at the time of the earthquake, so some specific time path of the building's market value -- a depreciation rate, v -- must be incorporated into the analysis. This, the cost-minimization problem is as follows:

$$(1) \min_x K(1+x) + \int_{t_0}^I \int_0^N K p(t) f(K, x, i) e^{-(r+v)t} q(i) dt di .$$

In this formulation, the frequency of earthquakes is assumed to be sufficiently low that no more than one will occur during the life of a building. Consequently, after an earthquake occurs, no additional value is derived from expenditures xK on seismic resistivity. Moreover, the term representing earthquake damage is simplified to represent the expected damage from precisely one possibility of an earthquake during

the N years of a structure's life.

The first order condition for cost minimization is:

$$(2) \quad K + K \int_0^N p(t) e^{-(r+v)t} dt \int_{i_0}^I f_x(K, x, i) q(i) di = 0 .$$

If the further assumptions are made that v can be approximated by $\frac{1}{N}$ and that the probability of an earthquake is uniformly distributed with respect to time, equation (2) with the integration performed and rearranged, reduces to:

$$(3) \quad 1 + p \left(\frac{1 - e^{-(r + \frac{1}{N})N}}{r + \frac{1}{N}} \right) \int_{i_0}^I f_x(K, x, i) q(i) di = 0 .$$

The integral $\int_{i_0}^I f_x(K, x, i) q(i) di$ is the expected value of f_x over i , and is denoted by $G(f_x(K, x, i))$.

In nearly all cases, the value of N is very large. Consequently, expression (3) can be simplified by use of the following approximation:

$$\left(\frac{1 - e^{-(r + \frac{1}{N})N}}{r + \frac{1}{N}} \right) = \frac{1}{r}$$

In the analysis to follow, this simplification will be used rather than estimating the true value of N and evaluating the complicated exponential.

To generalize the cost-minimization problem to account for external costs, another element is added to the optimization problem that represents the costs of earthquake damage that are not borne by owners of a structure. Two additional terms must be defined:

$E(K,i)$ = the external costs caused by ground shaking of intensity i at a structure of size K , if it were constructed without account being taken of earthquake risks;

$g(K,x,i)$ = the proportion of loss $E(K,i)$ that is suffered if xK is spent on defense.

The function $E(K,i)$ covers a multitude of damages, running from the easily monetizable (e.g. government disaster relief expenditures to clean up rubble) to the dubiously monetizable (e.g. pain, suffering and death of persons in or around the collapsed structure). Again for ease of exposition, these damages are assumed not to be time-dependent.

The external damage function is included here for the purpose of examining qualitatively its implications in the optimization problem, not because there is much hope of estimating the function empirically. Here $E(K,i)$ is assumed to rise as K increases because larger buildings effect more people and a wider area when they collapse; e.g. $E_K(K,i) > 0$.

The function $g(K,x,i)$ serves the same purpose as did $f(K,x,i)$ in the private cost-minimization problem. It is assumed that expenditures on defense against earthquakes reduce external costs and are more important in large structures. Moreover, because $g(K,x,i)$

$\in [0,1]$, the function must for some range become less sensitive to further increases in K and x . These requirements can be expressed as:

$$g_K \geq 0; \quad g_x \leq 0; \quad g_{KK} \leq 0 \text{ for } K \geq K^*; \quad g_{xx} \geq 0 \text{ for } x \geq x^*; \quad g_{xK} \leq 0.$$

The assumption that the same x enters $f(K,x,i)$ and $g(K,x,i)$ presumes an identity of actions to protect buildings and measures to protect people. While this may be generally correct, there may be exceptions. For example, flexible high-rise structures may be more resistant to seismic shocks than rigid buildings, but the people at the top of flexible buildings may be injured by being flung about as the structures sway in response to a seismic shock. The analysis in this paper ignores any potential conflicts between saving lives and protecting buildings.

Social welfare maximization entails including the external costs of an earthquake in the private optimization problem, thereby arriving at the following cost-minimization problem:

$$\min_x K(1+x) + \int_0^I \int_0^N \left[Kf(K,x,i)e^{-(r+v)t} + E(K,i)g(K,x,i)e^{-st} \right] p(t)q(i)dt di$$

where s is the social discount rate.

The second term in brackets represents the external costs of earthquakes, and the entire integrand represents the expected social loss in a time interval. The social discount rate, s , is not necessarily the same as the rate, r , used in the rest of the objective expression. Writing these rates as different parameters avoids taking an explicit stand on the appropriate discount rate to apply to irreversible events, such as the loss of life. Whether $s=r$, $s=0$, or $0 < s < r$ is not germane for the purposes of this paper because no attempt will be made to evaluate the social externalities associated with an earthquake.

As is readily apparent, the first-order condition for this problem contains the terms in the first-order condition for private cost minimization, plus an additional term reflecting external costs. Making the same assumptions as were made in the preceding analysis, this term reduces to:

$$\frac{p}{r} \int_{i_0}^I E(K,i) g_x(K,x,i) di,$$

in which the integral is the expected external damage if a damaging earthquake occurs, which is henceforth written as $H(E, g_x)$. The assumptions on functional forms imply that this expression will be negative as long as further effective earthquake defenses are available. Hence, the value of x that satisfies (3) (and thereby minimizes private costs) will be too small to minimize social costs. The equilibrium condition for minimizing social cost can be written as:

$$\frac{p}{r} [G(f_x) + H(E, g_x)] = 0. \quad (3)'$$

II. EMPIRICAL APPLICATION

The strategy of this section is to provide a crude index of the extent to which the expression for private profit maximization is in disequilibrium. If building codes require an investment in earthquake defenses that diverges from the optimal private value of x , then expression (3) will not be satisfied. However, expression (3) minimizes only private structural costs. If society is optimizing building codes, the disequilibrium will indicate the social valuation of the nonstructural costs of earthquake damage; that is, the actual value of $-\frac{p}{r} G(f_x)$ is the value that $\frac{p}{r} H(E, g_x)$ must take to minimize social costs. Thus, the extent to which the expenditures on defenses diverge from the cost-minimizing equilibrium provides a measure of the value of the external effects of earthquakes that is embodied in the codes, that is, an estimate of the social valuation of

$$\frac{p}{r} \int_{i_0}^I E(K,i) g_x(K,x,i) di$$

The desirability of a code can then be viewed as a subjective assessment of whether this implicit valuation of external effects is reasonable.

The empirical analysis to follow must be a crude approximation to reality. One source of difficulty is due to the fact that each structure is to some degree unique. The likelihood that a structure will collapse in an earthquake depends upon the location of the structure relative to the epicenter, the magnitude of the earthquake, the orientation of the axes of the structure in relation to the direction of ground motion due to the earthquake, the composition of

the ground on which the structure is built, and numerous other features that are specific to the site, the structure and the nature of the earthquake threat that it faces. The problem of designing an optimal degree of seismic resistivity into a structure is, therefore, unique to each building. A uniform building code is intrinsically inefficient in that it requires some degree of uniformity among structures that is not strictly desirable. As local government has become more sophisticated about the nature of the threat of earthquakes, building codes have begun to move in the direction of design standards based on the characteristics of a building site that affect its seismic vulnerability.^{-/} Nevertheless, given the costs of writing

^{-/} For example, in Long Beach, California, old commercial structures and apartment houses are assigned to a "seismic hazard" category. Each design feature that is related to seismic vulnerability is assigned a score, and the total score for the structure determines its risk category. Owners are given a fixed amount of time to improve the seismic resistivity of the structure or remove it, with the amount of time being shorter for greater risks. Owners can stretch this period by repairing part of the hazard and causing the building to move to a lower risk category.

and enforcing a unique code for each structure, the least costly strategy is likely to be to maintain considerable uniformity of treatment among buildings. In most localities, the movement away from uniform codes is only beginning. Consequently, an empirical

analysis must be a fictional representation of conditions for a "representative" structure.

Expression (3) has minimal data requirements. In order to determine whether equation (3) is satisfied for a particular type of structure, one need only know: (1) the annual probability of an earthquake, (2) the proportion of the area exposed to different amounts of ground shaking intensity, given a major earthquake, (3) the life of the structure if no earthquake occurs, (4) the long-term interest rate, and (5) the expected reduction in the proportion of damage to the investment due to a given intensity of ground shaking that was accomplished by the last percentage point increase in the fraction of building costs that is attributable to seismic resistivity.

Earthquake Frequencies and Intensities

One inescapable data requirement for estimating the appropriate amount of seismic defenses in the manner implied by the proceeding model is information about the frequency of damaging earthquakes. Perhaps surprisingly, little information is available about the frequency of earthquakes in specific locations. Damaging earthquakes are very rare events, even in seismically active areas, and meaningful data on the intensity of earthquakes has been collected only for about the last hundred years. Two measures of earthquake intensity are commonly used: Richter Magnitude (RM) and Modified Mercalli Intensity (MMI). The former is a measure of the energy released during the peak of an earthquake at its epicenter, and the latter is a crude categorization scale based upon the effects of the

earthquake at a particular point.

The Mercalli measure is made at each location affected by an earthquake, and varies from one location to another according to distance from the epicenter of the earthquake and ground conditions. Because the concern here is with structural damage, the MMI scale is the appropriate measure. That is, the function $p(t)$ will denote the frequency of earthquakes that measure at least some minimally damaging value of MMI. The maximum intensity of ground shaking due to an earthquake is commonly denoted by MMI_0 .

Table 1 shows the surprisingly vague standard definitions of the twelve categories of the Mercalli intensity. MMI VIII represents the threshold at which structural collapse of modern buildings occurs. As Table 5, below, shows, MMI VIII is also the minimum intensity at which any substantial number of modern buildings will suffer heavy damage. Accordingly, $p(t)$ will measure the frequency of ground shaking of at least MMI VIII.

TABLE 1

Modified Mercalli Intensity Categories

- I. Not felt except by a very few under especially favorable circumstances.
- II. Felt only by a few persons at rest, especially on upper floors of buildings. Delicately suspended objects may swing.
- III. Felt quite noticeably indoors, especially on upper floors of buildings, but many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibration like passing of truck. Duration estimated.

- IV. During the day felt indoors by many, outdoors by few. At night some awakened. Dishes, windows, doors disturbed; walls made creaking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably.
- V. Felt by nearly everyone; many awakened. Some dishes, windows, etc., broken; a few instances of cracked plaster; unstable objects overturned. Disturbance of trees, poles, and other tall objects sometimes noticed. Pendulum clocks may stop.
- VI. Felt by all; many frightened and run outdoors. Some heavy furniture moved; a few instances of fallen plaster or damaged chimneys. Damage slight.
- VII. Everybody runs outdoors. Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable in poorly built or badly designed structures; some chimneys broken. Noticed by persons driving motor cars.
- VIII. Damage slight in specially designed structures; considerable in ordinary substantial buildings with partial collapse; great in poorly built structures. Panel walls thrown out of frame structures. Fall of chimney, factory stacks, columns, monuments, walls. Heavy furniture overturned. Sand and mud ejected in small amounts. Changes in well water. Disturbs persons driving motor cars.
- IX. Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb; great in substantial buildings, with partial collapse. Buildings shifted off foundations. Ground cracked conspicuously. Underground pipes broken.

- X. Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations; ground badly cracked. Rails bent. Landslides considerable from river banks and steep slopes. Shifted sand and mud. Water splashed (slopped) over banks.
- XI. Few, if any, (masonry) structures remain standing. Bridges destroyed. Broad fissures in ground. Underground pipe lines completely out of service. Earth slumps and land slips in soft ground. Rails bent greatly.
- XII. Damage total. Waves seen on ground surfaces. Lines of sight and level distorted. Objects thrown upward into the air.

Source: S. T. Algermissen, et al., A Study of Earthquake Losses in the Los Angeles, California Area, U.S. Government Printing Office, 1973.

The frequency of intense ground shaking due to earthquakes varies according to location, and so empirical application of the general method developed in this paper must be site-specific. The data used here are from California where information about earthquakes has been systematically gathered for the longest period of time.—/

/ In part, California has better data because it is in an active seismic region, but this is not the only explanation. Major earthquakes are roughly as great a threat in several other parts of the United States -- notably Alaska, parts of the Rocky Mountain Area, Missouri, Tennessee, and Eastern Massachusetts. Another factor contributing to the richer data base in California is the

extent and quality of research on seismology at the U.S. Geological Survey offices in California and at the state's major universities, notably Caltech, U.C. Berkeley and Stanford. The modern science of geophysics was, for the most part, invented in California, and California had the first extensive seismic network for measuring earthquakes.

The estimate of $P(t)$ is necessarily crude, because data on major earthquakes are extremely scarce, even in California. The lowest magnitude Southern California earthquake which has a recorded intensity of MMI VIII measured 5.8 on the Richter scale.—/ Earthquakes of roughly

/ S. T. Algermissen, et al., A Study of Earthquake Losses in the Los Angeles, California Area, U.S. Government Printing Office, 1973.

this magnitude occur about every two years, but most often are centered in unpopulated areas and, therefore, cause no significant damage. Great earthquakes causing widespread damage are sufficiently infrequent that few have been observed. But this does not make the major earthquakes necessarily of lesser importance. The more intense the earthquake, the larger the area affected by it. A magnitude 6.0 earthquake will cause severe ground motion and serious structural damage in an area with a radius of about fifteen miles, but is unlikely to cause structural collapse of modern buildings; the area of damaging shaking has a radius of about 50 miles for a magnitude 7.0 earthquake and of about 90 miles for a magnitude 8.2, although in both instances the area in which structural collapse will occur is much smaller.—/ Housner has

/ G. W. Housner, "Vibration of Structures Induced by Seismic Waves:

Part I - Earthquakes," in M. Harris and E. Crede, Shock and Vibration Handbook Vol. 3, McGraw-Hill, 1961.

estimated the frequency of earthquakes in California by Richter magnitude and the area of severe shaking associated with each magnitude. The results are summarized in Table 2. The Housner radius includes shaking of intensity MMI VII or greater.

TABLE 2

Probable Number of Earthquakes and Area of Severe Ground Motion Per Century by Richter Magnitude for State of California

Richter Magnitude	Number of Earthquakes per 100 Years	Area in Square Miles Subject to Severe Ground Shaking per Earthquake	Total Area Affected by all Earthquakes of Magnitude Class
6.0 - 6.3	46	1,000	46,000
6.4 - 6.7	27	3,000	81,000
6.8 - 7.3	19.3	8,000	154,000
7.4 - 7.9	5.6	15,000	84,000
8.0 - up	1.1	25,000	28,000

Source: Based upon data and calculations in G. W. Housner, ibid.

Using these data and Housner's calculation regarding the area affected by earthquakes of varying magnitudes, the size of the areas affected by earthquakes of each range of magnitudes can also be estimated, as is shown in Table 2. The last column sums to

approximately 400,000 square miles; considering that California contains approximately 150,000 square miles and that nearly all of the state is seismically active, these data imply that a particular seismically active location can expect to experience severe ground shaking about once every thirty to forty years.

Ground shaking of a severity postulated in the calculations shown in Table 2 does not necessarily lead to structural collapse. Table 3 compares RM and MMI for 23 Southern California earthquakes that occurred from 1907 to 1973, for which estimates of both intensity measures have been made. The Housener calculation is intended to include areas of

TABLE 3

Relationship Between Richter Magnitude and Modified Mercalli Intensity: 1907-1973

Range of Magnitude	Number of Earthquakes	Number with MMI VIII or above
5.8 - 6.1	11	3
6.2 - 6.5	8	4
6.6 - up	4	4

Source: S. T. Algermissen, et al., A Study of Earthquake Losses in the Los Angeles, California Area, U.S. Government Printing Office, 1973.

serious structural damage that falls short of collapse, as is evident by the inclusion of magnitude 6.0 - 6.3 earthquakes in the calculations. These earthquakes are rarely associated with the collapse of modern structures, or with large nonstructural costs, as is shown below.

Wiggins and Moran have presented a formal representation of the relationships among Mercalli Intensity, Richter magnitude and the distance (r) from the epicenter of the earthquake as follows:—/

$$\text{MMI} = 4.7 + 1.04\text{RM} - 3.14 \log r.$$

—/ J. H. Wiggins and D. H. Moran, Earthquake Safety in Long Beach Based on the Concept of Balanced Risk, J. H. Wiggins Co., 1971 (mimeo).

This equation should not be taken too seriously. Wiggins and Moran provide no justification for the functional form nor any measures of the statistical significance of either the individual variables or the equation as a whole. Thus, whether the relation is even remotely accurate for large earthquakes, which lay at the extreme values covered by the data being analyzed and which necessarily constitute a small part of the sample owing to their infrequency, remains in doubt but seems unlikely. In any event, the equation suggests that the MMI VII region for a magnitude 8.0 earthquake constitutes about half of the total area that suffers significant damage. The implication is that less than half of the areas included in the calculations in Table 3 are shaken severely enough so that heavy structural damage or collapse is a possibility. If so, the frequency of shaking of intensity MMI VIII or greater at a given location is half the frequency of major earthquakes, or, at most, once every sixty to eighty years. Thus, in estimating expression (9), values of p less than .02 are appropriate.

Values of q(i) are even more uncertain than for p(t). Table 2,

the Wiggins-Moran relation, and other data on the relative frequencies of various degrees of shaking intensity suggest that for earthquakes that may cause structural collapse, more than half the area shaken is subject to MMI VIII and about one-fourth to MMI IX. The remainder is subject to more severe ground shaking.

Relating Earthquake Intensity to Damages

An estimate of the proportion of the investment lost due to ground shaking is dependent on the nature of the structure. Whitman, et al., have estimated the probabilities of varying degrees of damage to 8-13 story structures of a particular design, given the Mercalli intensity of the earthquake at the site of the structure. Table 4 defines the damage categories used by Whitman, et al.

TABLE 4
Relationship of Structural Damage to Death and Injury

Damage State	Fraction of Structure Lost	Fraction of Occupants Killed	Fraction of Occupants Injured
Moderate	.05	0	.01
Heavy	.30	.0025	.02
Total	1.00	.01	.10
Collapse	1.00	.20	1.00

Source: R. V. Whitman, J. M. Biggs, J. E. Brennan III, C. A. Cornell, R. L. deNeufville and E. H. Vanmarcke, "Seismic Design Decision Analysis," Journal of the Structural Division, American Society of Structural Engineers, May 1975, p. 1077.

Of course, because larger buildings are more vulnerable to earthquakes, the figures in Table 4 would be substantially smaller for one or two story structures. An important element of this table is its demonstration of the fact that the damage to a structure must be fairly severe before many of the occupants of the structure are likely to be hurt.

Table 5 shows the effect on the probability that the hypothetical 8-13 story structure will suffer each damage state of different values of the Mercalli Intensity of the earthquake and differences in the seismic resistivity of the structure. Three seismic codes are shown: UBC 0-1 corresponds to the construction of modern buildings without reference to the seismicity of the site; UBC 3 corresponds to the codes now in force in most localities in which the threat of earthquakes is greatest within the United States, such as California; and UBC S represents new standards that have been developed during the 1970s that would make buildings more resistant to earthquakes than do the old UBC 3 codes.

Evaluating Seismic Codes

In order to estimate the value of $\int_{i_0}^I f_x(K, x, i) q(i) di$
 $= G(f_x(K, x, i))$ in expression (3), information must be obtained on the changes in expected losses for each relevant intensity level, i , and in construction costs due to a change in building codes. The ratio, $\frac{\Delta f}{\Delta x}$, of these two changes can be used to estimate $G(f_x(K, x, i))$.

TABLE 5
Probability of Damage State for Various
Earthquake Intensities and Seismic Building Codes

Code	Damage State	Mercalli Intensity of Earthquake				
		MM VII	MM VIII	MM IX	MM X	MM XII
UBC 0-1	Moderate	.33	.20	0	0	0
	Heavy	.04	.41	0	0	0
	Total	0	.34	.75	.25	0
	Collapse	0	.05	.25	.75	1.00
UBC 3	Moderate	.25	.53	.20	0	0
	Heavy	0	.21	.52	0	0
	Total	0	.01	.23	.80	0
	Collapse	0	0	.05	.20	1.00
UBC S	Moderate	.21	.52	.30	0	0
	Heavy	0	.08	.58	0	0
	Total	0	0	.02	.90	0
	Collapse	0	0	0	.10	1.00

Source: Whitman, et al., p. 1077.

Tables 2, 4 and 5 provide the information necessary to estimate the change in expected losses. Multiplication of the damage estimates for each type of structure in Table 4 and the probabilities of damage states, given the occurrence of an earthquake, in Table 5, produces the expected loss from each intensity of earthquake for each type of structure. If these entries are then multiplied by the relative frequencies of each intensity level of earthquake and the products are summed for each category of structure, the results are the expected damage to each type of structure, conditional on the occurrence of a damaging earthquake. The value to a building owner of a movement from UBC 3 to UBC S, which is the relevant choice in most communities in seismically active areas, can be estimated by comparing these expected losses. The result of these calculations is that the change in building codes makes a measurable difference to building owners only in the two situations in which the code substantially affects the probability of total loss: MMI VIII and MMI IX. The expected loss under UBC S differs from the loss under UBC 3 by 5 and 24 percent, respectively, for these two intensities. According to the preceding calculations on the likelihood of severe earthquakes, the relative frequencies of these states are one-half for MMI VIII and one-fourth for MMI IX. Thus, the expected gain to a building owner of building to UBC S instead of UBC 3 is approximately 9 percent of the value of the structure, conditional on the occurrence of an earthquake that produces MMI VIII or higher shaking at the site. Of course, society would derive additional benefits from this change in codes. The most important element of the additional social gain is the increase in the

proportion of buildings that are destroyed but that do not collapse, as building collapse is the principal threat to life as well as the principal disrupting event to the normal life of the community.

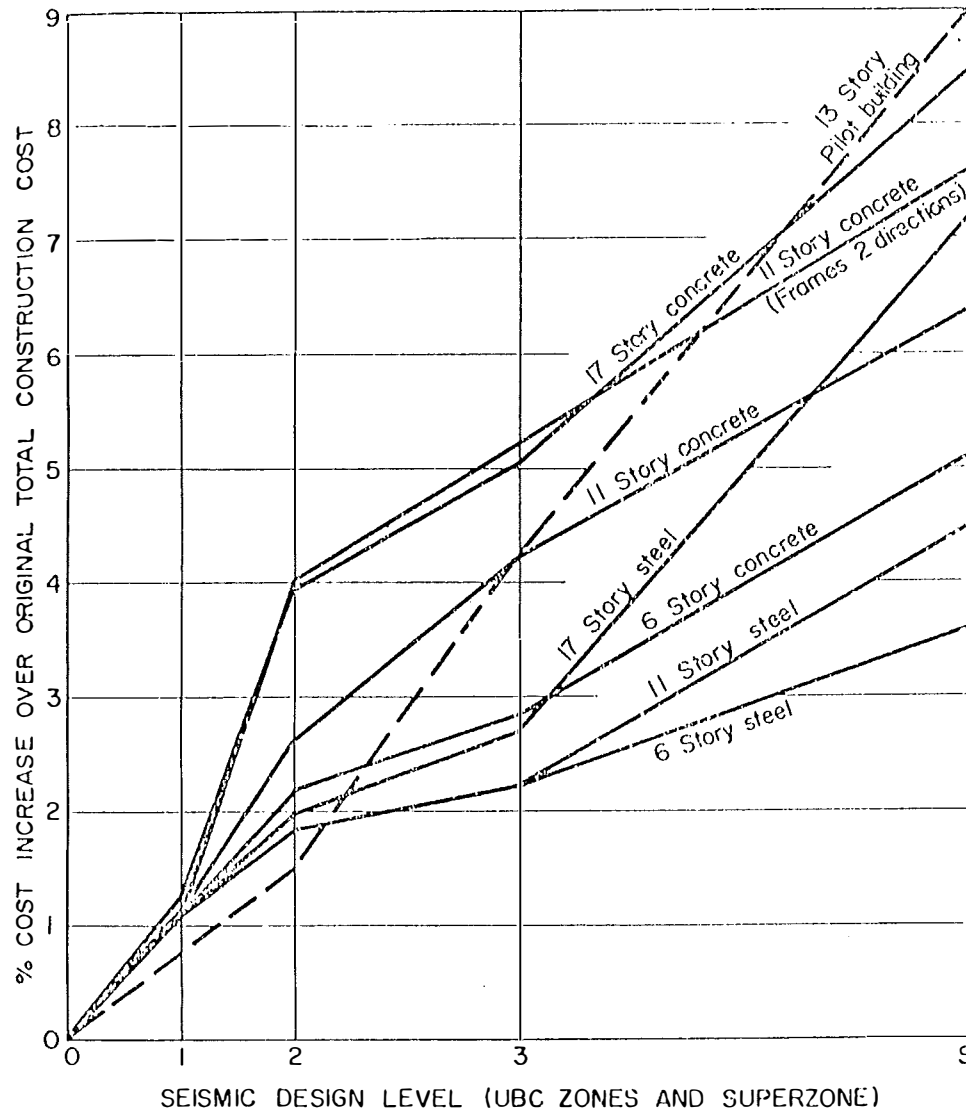
The second element entering into the estimate of $G(f_x(K, x, i))$ is the cost of adopting UBC S compared to UBC 3. Whitman, et al have performed an engineering design study of the cost of constructing buildings to satisfy various building codes, and their results are summarized in Figure 1. As the figure makes clear, moving from UBC 3 to UBC S increases the cost of a structure by 1 to 5 percent, depending upon the type of structure. This neglects the value of the contents of the structure, which is ignored here.

Based upon these calculations, in the range of UBC 3 and UBC S, the value of $G(f_x(K, x, i))$ can be placed within the range of -2 and -9. Armed with this information, expression (3) can now be crudely estimated.

Table 6 shows calculated values of the left side of (3) based on several values of p , $G(f_x)$ and r that are possible candidates for the "correct" value for a particular structure. Comparison of the results for different values of these parameters provides a sensitivity analysis of the model. At the point of minimization of expected private costs, (3) would equal zero. Positive entries in the matrix correspond to combinations of values of the variables in (3) that indicate that UBC S goes beyond the degree of seismic resistivity that minimizes the private cost function that includes seismic risks. Negative entries correspond to situations in which even UBC S falls short of the best design to protect the private investment in the structure.

Figure 1

Costs of Alternative Seismic Resistivity Codes



Source: R. V. Whitman, J. M. Biggs, J. E. Brennan III, C. A. Cornell, R. L. deNeufville, and E. H. Vanmarcke, Methodology and Pilot Application: SDDA Report No. 10, Department of Civil Engineering, Massachusetts Institute of Technology, 1974 (mimeo).

TABLE 6

$$\text{Values of } 1 + \frac{p G(f_x(K, x, i))}{r}$$

	$r = .05$					$r = .10$				
	$p = .1$	$p = .02$	$p = .01$	$p = .005$		$p = .1$	$p = .02$	$p = .01$	$p = .005$	
$G(f_x) = -1$	-1.0	.6	.8	.9		0	.8	.9	.95	
$G(f_x) = -2$	-3.0	.2	.6	.8		-1.0	.6	.8	.9	
$G(f_x) = -4$	-7.0	-.6	.2	.6		-3.0	.2	.6	.8	
$G(f_x) = -7$	-13.0	-1.8	-.4	.3		-6.0	-.4	.3	.65	
$G(f_x) = -10$	-19.0	-3.0	-1.0	0		-9.0	-1.0	0	.5	
$G(f_x) = -50$	-29.0	-5.0	-2.0	-.5		-14.0	-2.0	-.5	.25	

Source: Calculations in Text.

For the pilot building in the Whitman, et al. study, which was the basis for calculating Tables 4 and 5, the costs of moving from UBC 3 to UBC S are about 5 percent of total costs so that the value of $f_x(K, x)$ is approximately -1.8. For a discount rate of .10 and a probability equal to .01 that shaking of MMI VIII or greater will occur, the value of (3) is .8. More generally, realistic values of the variables -- e.g. (f_x) less than 9, r near .10 or higher, and p less than .02 -- produce a pattern of positive entries.

This means that UBC S involves more resistivity than owners of structures would freely pick -- a happy result in terms of verifying the model used herein, because owners are now observed rarely to adopt UBC S on their own.¹ These results also confirm

¹ Whitman, et al. (1975), note that the time of the 1971 San Fernando earthquake, the Los Angeles building stock was vulnerable because most buildings were not being built to this standard. They also make a similar observation about Boston.

the view that the principal purpose of codes is to internalize external effects, rather than to offset irrationality or ignorance. One can readily compute the value of the external costs that are implicit in UBC S. In order for (3)' to be zero (given r and p), $G(f_x) + H(K, g_x)$ would have to be -10. For this to be the case, the costs of moving from UBC 3 and UBC S would have to total .9 of a percent of the costs of the structure rather than about 5 percent. To force the pilot building to satisfy UBC S is, implicitly, to

value the avoidance of damage other than to the structure that UBC S makes possible at about 4.5 times the value of the savings to the building from adopting UBC S. According to the calculations based on Tables 4 and 5, the latter is approximately 9 percent, so the bottom line of the calculations is that, given an earthquake of damaging intensity, the savings from UBC S other than to the owner must equal about 40 percent of the value of the building.

The preceding calculations are offered as illustrations of the uses to which the analysis and data summarized in this paper can be put. Forty percent of the cost of a thirteen-story building may or may not be a good estimate of the savings in terms of external costs that would accrue, given a damaging earthquake, if the buildings were built to UBC S specifications instead of UBC 3. Whatever the answer to this loaded question, the type of analysis provided in this paper provides a mechanism for collapsing the debate about seismic building codes into the rather simple framework of whether the expected savings are worth the cost. Moreover, the process of making these calculations is instructive because it encounters many relatively difficult data problems along the way. In particular, information on the underlying events that give rise to seismic codes -- namely, the intensity of shaking from earthquakes that can be expected during the life of a structure -- is too poor to make more than crude empirical efforts possible.

III. THE EFFECTS OF EARTHQUAKE PREDICTIONS

In this section a more general investment problem is analyzed,

that of maximizing the (nonzero) returns of a capital investment. The asset is threatened by a random event that, if it occurs, will reduce the earnings potential of the asset. This formulation is not useful for empirical purposes, because it assumes data regarding the expected revenue stream from a structure as a function of the amount of investment made in it. A more general formulation is valuable for deriving theoretical results. The model formulated here is used to investigate changes in investment patterns that will result from improved earthquake prediction technology.

Define $R(K)$ to be the annual income earned from an investment K if the structure has not yet been damaged by an earthquake.—/

—/ The revenue may depend on x as well, assuming renters can tell the seismic resistivity of a structure by examining it. Making x an argument of R greatly complicates the manipulations to follow, but it does not change the qualitative theoretical results or the implications for empirical work.

The revenue function, $R(K)$, is assumed to exhibit positive marginal revenue of capital ($R_K > 0$) for at least some $K > 0$, but with declining marginal revenue for all values of K (e.g. $R_{KK} < 0$). Revenues are assumed to be the same in each year, an assumption that greatly simplifies the theoretical exposition. The revenue function is not assumed to be linear, for locational factors are likely to produce an optimal K for each site. Of course, this does not imply

noncompetitive behavior — for each K , the builder can be a price taker, but face a given functional relationship between K and R that allows a single choice of K and x that produces nonnegative profits. Let $F(t)$ be the probability that an earthquake that would cause structural collapse of $x = 0$ will occur by time t in the area of seismic risk in which the structure is located, with time $t = 0$ the date of construction of the structure, on the notation in Section I, $p(t) = F'(t)$.

A convenient minimum value, m^* , for the magnitude of earthquakes considered here is one which causes some modern buildings to suffer collapse if they are not constructed to withstand seismic shocks, i.e., if $x = 0$. The same earthquake causes moderate to heavy damage in some of the structures further from the center of the earthquake fault. The probability density function (p.d.f.), $q(i)$, distinguishes between areas subject to different degrees of ground shaking. If $F(t, m)$ is the cumulative distribution function (C.D.F.) from time zero to time t of an earthquake of magnitude $m \geq m^*$, and if $q(i|m)$ is the conditional p.d.f. of intensity $i \geq i_0$, given a magnitude m earthquake, then:

$$F(t) = \int_{m \geq m^*} F(t, m) dm, \quad \text{and}$$

$$q(i) = \frac{\int_{m \geq m^*} q(i|m) F(t, m) dm}{\int_{m \geq m^*} F(t, m) dm}$$

Within this framework, the profit-maximization problem of a private investor contemplating an investment in a building is:

$$\begin{aligned} \text{Max}_{K,x} \quad & \int_{t=0}^N R(K) e^{-(r+v)t} dt - (1+x)K \\ & - \int_{i=i_0}^I \int_{t=0}^N R(K) f(K, x, i) F(t) q(i) e^{-(r+v)t} dt di. \end{aligned}$$

The first term is the discounted present value of the stream of revenues from the structure, the second term is the total cost of the structure (including defensive costs built into the building), and the third term represents the discounted present value of expected revenue loss due to earthquakes.

The necessary conditions for a maximum for this profit expression are:

$$\begin{aligned} (4) \quad & R'(K) \left(\frac{1 - e^{-(r+v)N}}{r+v} \right) - (1+x) \\ & - \int_{i=i_0}^I \left\{ \left[R'(K) f(K, x, i) + R(K) f_K(K, x, i) \right] q(i) \int_{t=0}^N F(t) e^{-(r+v)t} dt \right\} di \\ & = 0. \end{aligned}$$

$$(5) \quad K + \int_{i=i_0}^I \left\{ R(K) f_x(K, x, i) q(i) \int_{t=0}^N F(t) e^{-(r+v)t} dt \right\} di = 0.$$

The integral of the discounted C.D.F. $\left(\int_{t=0}^N F(t) e^{-(r+v)t} dt \right)$ will be written as J . The second order conditions are as follows:

$$\begin{aligned} (6) \quad & R''(K) \frac{1 - e^{-(r+v)N}}{r+v} \\ & - J \int_{i=i_0}^I \left\{ R''(K) f(K, x, i) + 2R(K) f_{KK}(K, x, i) + R(K) f_{KKK}(K, x, i) \right\} q(i) di \\ & \leq 0. \end{aligned}$$

$$(7) \quad J \int_{i=i_0}^I R(K) f_{xx}(K, x, i) q(i) di \leq 0.$$

$$(8) \quad 1 + J \int_{i=i_0}^I \left\{ R'(K) f_x(K, x, i) + R(K) f_{Kx}(K, x, i) \right\} q(i) di < 0. \quad \text{—/}$$

/ The actual condition on this term is that it be less than a strictly negative term involving expressions (6) and (7); however, only the weaker version in (8) will be used below.

These conditions, especially (4) and (5), permit some exploration of the effects of the introduction of prediction technology by use of a conventional comparative statics analysis. A valid earthquake prediction can be expressed as an increase in $F(t)$, and hence J , for values of t near zero. Conversely, the absence of a prediction supports the negative inference that, for t near zero, $F(t)$ is less than the historical frequency of seismic shocks.

Total differentiation of (4) and (5) produces the following conditions:

$$\begin{aligned}
 (9) \quad & \frac{dx}{dJ} \left[1 + J \int_{i_0}^I \left\{ R'(K) f_x(K, x, i) + R''(K) f_{Kx}(K, x, i) \right\} q(i) di \right] \\
 &= \frac{dx}{dJ} \left[R'(K) \frac{1 - e^{-(r+v)N}}{r+v} - J \int_{i_0}^I \left\{ R''(K) f(K, x, i) + 2R'(K) f_{Kx}(K, x, i) \right. \right. \\
 &\quad \left. \left. + R(K) f_{KK}(K, x, i) \right\} q(i) di \right] \\
 &- \left[\int_{i_0}^I \left\{ R'(K) f(K, x, i) + R(K) f_{Kx}(K, x, i) \right\} q(i) di \right].
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \frac{dx}{dJ} (R(K) J \int_{i_0}^I f_{xx}(K, x, i) q(i) di) \\
 &= \frac{-dx}{dJ} \left(1 + J \int_{i_0}^I \left\{ R'(K) f_x(K, x, i) + R(K) f_{xK}(K, x, i) \right\} q(i) di \right) \\
 &\quad + R(K) \int_{i_0}^I f_x(K, x, i) q(i) di. \quad \text{—/}
 \end{aligned}$$

/ It is assumed that $q(i)$ is insensitive to changes in J , e.g. a positive prediction raises the probability of all large earthquakes occurring.

Given the current state of knowledge on the relation of intensity and ground-shaking for very large earthquakes, the total area

affected increases with magnitude but the ratio of land suffering heavy damage to land suffering total damage remains roughly the same (see Section II). However, a prediction might narrow the variance of the expected magnitude of the earthquake. If it predicts an increased chance of a major earthquake but rules out a great earthquake, $\frac{dq}{dJ}$ might be negative. The left sides of both (9) and (10) then include the term

$$- J R(K) \int_{i_0}^I f_x(K, x, i) \frac{dq}{dJ}(i) di$$

which is indeterminate in sign.

From the second-order conditions and the assumptions about the functional forms of $R(K)$ and $f(K, x, i)$, all of the terms (9) and (10) can be signed, and the following established.

Proposition 1: If an earthquake is predicted (i.e., $dJ > 0$), the optimum expenditure on capital structures, K , will decline and the optimum expenditure proportion devoted to protection, x , will increase; conversely, the absence of a prediction will have the opposite effects.

The invention of reliable prediction technology will, then, have an indeterminate effect on the safety of buildings, depending on the form predictions take and the frequency of prediction. At one

extreme, predictions might be certain, and may foresee some fraction, α , of all earthquakes. Because earthquakes that affect any given locality are extremely rare events, in nearly all years this prediction technology would forecast no event.

In these years, the expected probability of a damaging earthquake would be $(1-\alpha)h$, where h is the historical frequency, and $F(t)$ would be correspondingly reduced. Only rarely would $F(t)$ be increased in the immediate years, and so, according to Proposition 1, the net effect over time of the introduction of prediction technology would be to reduce the strength of most structures.

At the other extreme, a prediction technique might detect conditions that are only weakly associated with earthquakes but that occur frequently. Letting δ represent the increment to h that makes $(h+\delta)$ be the probability of a damaging earthquake when the condition is present, as long as the condition is observed sufficiently frequently the impact of the prediction could be to raise the mean strength of buildings, depending on the exact shape of $R(x)$ and $f(K,x,i)$.

Proposition 1 holds for any source of change in the value of J . In addition to changes in the C.D.F. of major earthquakes, J also depends on N (the life of the structure) and r (the interest rate). Thus, by the same reasoning applied to changes in $F(t)$ due to predictions, the following proposition can also be established (as well as its converse).

Proposition 2: A fall in the interest rate or an increase in the durability of structures that is unrelated to seismic resistivity will lead to an increase in defenses against earthquakes.

Table 6 in Section II provides some further insight into the consequences of long-term earthquake predictions. If r is .10 and $G(f_x(K,x,i))$ is -2, consider the effect of a prediction that an earthquake was certain to occur in, say, the next ten years. In this case, p would equal .1, and the value of expression (3) would become strongly negative. In fact, for any time horizon under twenty years, the UBC S code would become less stringent than the standard that would minimize private costs. Alternatively, suppose a prediction technology indicates that for the next decade the chance of an earthquake has been halved, from a normal .01 to .005. Because of discounting, this eliminates most of the present-value benefits of a stronger structure, and would cause UBC S to become even less attractive to owners of buildings. Indeed, requiring UBC S in the face of such a prediction, given $r = .10$ and $g(f_x) = -2$, implies a social benefit equal to about nine times the expected benefits to the structure, rather than the fourfold relationship that would justify UBC S without a prediction.

IV. CONCLUSIONS

By casting the problem of optimal seismic resistance as a standard problem of optimal investment under uncertainty, two useful results have emerged. First, a method has been developed that enables

the analyst to use engineering cost information and other available data to estimate the minimum social benefits that a strengthening of the code must provide to be worthwhile. Second, the same method permits an estimation of the effects of changes in information about the frequency of earthquakes, such as valid predictions, on the private choice of seismic resistivity and on the implicit social benefit that justifies a particular code. Because of negative inferences from the absence of predictions, the introduction of a prediction method has an indeterminate but potentially important effect on incentives to defend against seismic shocks.